

Binding vs. scattering length: η in light nuclei

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Abstract

The possibility of etamesic nuclei remains an open problem in nuclear physics until now. Various calculations give contradictory predictions even for the lightest real nucleus ^3He . In this paper we present the connection of the binding energy and width to the complex scattering length for heavier nuclei than this in the hope that, with knowledge of the final state interaction this could be useful in searches of possible bound states.

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1 Introduction

In many unstable hadronic systems perhaps the only way to get information on their structure and interactions is the final state interaction in their formation process and associated decays, enhancement or decrease *vs.* free undistorted final state. Production slightly above the free threshold can yield also information on possible bound states below the threshold, especially if they are close to the threshold, *i.e.* weakly bound. This can be seen in the energy dependence [1,2] and described by the final state low-energy scattering parameters.

However, the cross section alone cannot distinguish whether the interaction can or cannot support a bound state. A textbook example is singlet S -wave NN scattering. It was necessary to indulge the difference in coherent neutron

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scattering off para- and orthohydrogen molecules to extract the sign of the large scattering length, which in turn showed that the interaction is not binding [3]. In most systems, in particular in case of spin-0 particles, this kind of extra information is not available. However, as final state interaction analyses anyway give *some* information, it is conceivable that this would be useful in experimental searches for bound or quasibound states. In the latter case the problem can be far from trivial, since the state, even if "bound", can be wide and correspondingly the low-energy scattering parameters would be complex.

An example of recent interest is the possibility of η -nuclear bound states. Numerous calculations exist, which disagree with each other completely especially for the lightest "real" nuclear systems with ^3He and ^4He [4–11]. Some of them indicate binding whereas most don't, while a general consensus is that by carbon binding should exist. Ref. [12] presents an overview of the confusion and a new fit for $(|a_R|, a_I)$ summarizing the experimental efforts.

Only a few experiments have been performed. One class of experiments produces the η -meson at rest in a quasi-free transfer reaction. In a second step the η interacts with a nucleon thus forming a resonance $N^*(1535)$ which can decay back to its entrance channel or, with 50% probability, into a nucleon and a pion. Since the η is at rest, these two final state particles are emitted almost back to back. The experiment by the GEM collaboration [13] claimed a 5σ effect in studying the $p+^{27}\text{Al}\rightarrow^3\text{He}+\pi^-+p+X$ reaction at a beam momentum for which the intermediate $X = \eta+(A-2)$ is almost at rest. In an experiment employing photoproduction the existence of η -mesic ^3He was claimed to have been observed in the reaction $\gamma^3\text{He}\rightarrow\pi^0+p+X$ using the photon beam at MAMI [14]. Similarly as in the previous experiment a two step process was assumed but only the pion was measured and not the other nucleons. It has, however, been pointed out [15] on the basis of new high statistics data for the excitation function of the reaction $\gamma+^3\text{He}\rightarrow\pi^0+p+X$ that the data of Ref. [14] do not permit an unambiguous determination of the existence of a $^3\text{He}\eta$ -bound state, because nucleon resonances produce opening angle dependent structures in excitation functions and subtraction of excitation functions for different opening angles can produce artificial structures almost anywhere.

Inclusive experiments searching for η -mesic nuclei at BNL [16] and LAMPF [17] by using a missing-mass technique in the (π^+, p) reaction reached negative or inconclusive results. Later it became clear that the peaks are not necessarily narrow and that a better strategy of searching for η -nuclei is required as for instance applied in Ref. [13]. Furthermore, the BNL experiment was in a region far from the recoilless kinematics, so the cross section is substantially reduced [18].

Another class of experiments searched for η -mesic nuclei in final state inter-

action. Intensive studies were dedicated esp. to the $p + d \rightarrow \eta + {}^3\text{He}$ reaction [19–21]. The $\eta^4\text{He}$ final state was studied in $d + d$ interactions making use of unpolarized beams [22–24] as well as polarized beams [25]. The heaviest system studied so far is $\eta^7\text{Be}$ produced in $p + {}^6\text{Li}$ reactions [26, 27].

With reasonable assumptions of the Watson-Migdal theory [1, 2] final state studies can give estimates for the imaginary value of the scattering length and the absolute value of its real part [12]. However, the sign of the latter would be crucial as a tell-tale of a bound state. Still, even $|a_R|$ could give indications of the value of the binding energy, *provided it exists*, useful for experiments searching for such states. Further useful information would be expectations of the width of such states. The aim of the present paper is to continue to heavier light nuclei the investigation of Ref. [28] for ${}^3\text{He}$ on the relation between binding and the low-energy scattering parameters.

The paper presents the minute amount of formalism next and then the results for representative mass distributions of three light nuclei.

2 Formalism

There is not much actual formalism in this paper. Rather the aim is numerical. The basic idea is to start from a simple optical model with a potential proportional to the density profile of the nucleus, use it to calculate the complex binding energy and scattering parameters separately and combine them to a common contour plot in the (a_R, a_I) plane. This presentation of the binding energy and width as a function of the complex scattering length is not necessarily trivial. Also with this phenomenology the validity of the simplest impulse approximation in the optical model.

For specificity (and to facilitate a comparison to the impulse approximation), the potential can be expressed as

$$V_{\text{opt}} = -4\pi(V_R + iV_I)\rho(r)/(2\mu_{\eta N}), \quad (1)$$

with $\mu_{\eta N}$ the reduced mass of the ηN system. Here the nuclear density ρ can be varied from nucleus to nucleus and for each nucleus the strength parameters are freely varied to get a sufficient coverage of the (a_R, a_I) plane. It should be stressed that we are not predicting any absolute strength of the potential as in the model works referred above. The main thing is the numerical connection of binding energies and widths to the scattering parameters, so that if the latter can be extracted from data, then a preliminary estimate could be obtained for the former. Although both are in some ranges sensitive to the potential parameters, in the spirit of the shape independence of NN forces, one might

expect a relatively density profile independent connection. Indeed, in Ref. [28] it was checked that the relation between (V_R, V_I) and (a_R, a_I) was robust against changes in the density profile. In contrast to the free variation, within the optical model (as *e.g.* in Ref. [4]) the strength would be related to the elementary ηN scattering length as $V_{R(I)} = A a_{\eta N, R(I)}$ with A the atomic number of the nucleus.

The scattering program is fairly standard even with a complex potential. The binding solutions are obtained searching by iteration for poles in the homogeneous integral equation equivalent to the Schrödinger equation. The convergence was good except for real potentials with very small binding energy (≤ 0.1 MeV) where the wave functions are much more extensive than the potential range. This case could reasonably be considered as essentially the zero binding limit with also extremely large cross section. Convergence stopped also in case of very large widths ($\Gamma/2 \geq 250$) MeV with the wave functions of shorter range than the potential. The latter case is certainly not of experimental interest (with binding still at most in low tens of MeV or rather a few MeV).

The scattering parameters are defined as is standard for mesons by

$$q \cot \delta = \frac{1}{a} + \frac{1}{2} r_0 q^2 \quad (2)$$

so that $a_R < 0$ means binding (we shall later bring up a more exact condition). Experiments extract so far only the scattering length a , but it is notable that the effective range r_0 is of the same order in the range of most interest.

3 Results

As a representative example the most detailed discussion is given to ^{12}C where binding is unanimously assumed. For this the modified harmonic oscillator of Ref. [29] is used as the density profile

$$\rho(r) = 0.0141 [1 + 1.15 (\frac{r}{1.672})^2] \exp [-(r/1.672)^2] \quad (3)$$

with the normalization $4\pi \int_0^\infty \rho r^2 dr = 1$.

The basic results are given in Figs. 1 and 2, where the binding energies (defined as $E_R < 0$) and half-widths $E_I = \Gamma/2$ are presented as contour points for 1, 2, 5, 10 and 20 MeV. The basic criterion for a printed point is that the deviation from the value is less than 0.05 MeV, though also a linear interpolation or extrapolation has been used in some more sensitive instances.

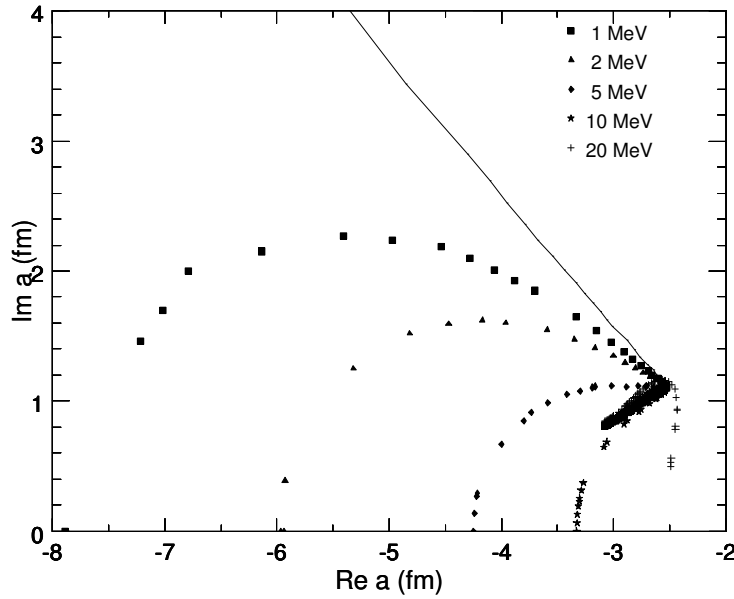


Fig. 1. The binding energy (E_R contours for 1, 2, 5, 10 and 20 MeV in the complex (a_R, a_I) plane). The line shows the zero energy, *i.e.* above it there is no binding as explained in the text.

At least for real potentials and small binding in Fig. 1 the results follow well the trend $\sim E_B^{-1/2}$ dictated by general arguments [30,31]. In fact, starting from the defining equation (2) one can derive a relation between the binding energy and low-energy parameters [32]. This can be generalized to the complex case as

$$1/a = -\sqrt{-2\mu_{\eta A}E - r_0\mu_{\eta A}E} \quad (4)$$

with $\mu_{\eta A}$ the reduced mass of the system. This relation was found to be amazingly accurate predicting the value of a well for binding energies up to $|E| \approx 10$ MeV and even beyond (a few percent for the real part and about ten percent for the imaginary, E and r_0 taken from the calculation).

Furthermore, it may be noted that for numerically bound points the condition [12]

$$\mathcal{R}[a^3(a^* - r_0^*)] > 0 \quad (5)$$

was well satisfied, while the simpler rule $|a_R| > a_I$ without the effective range, given *e.g.* in [11], was routinely broken for (wide) virtual states above threshold (*i.e.* the inequality was satisfied also for unbound states). However, even the condition (5) turns out to be a necessary condition but not sufficient. Although the expression does decrease by an order of magnitude for decreasing E_R for each given V_R , it could remain positive beyond the bound region.

With complex potentials the dependencies become nontrivial. As befits strong interactions, the strength of the potential V (real or imaginary) does not go to E or a linearly. However, in the former case the dependence of both E_R

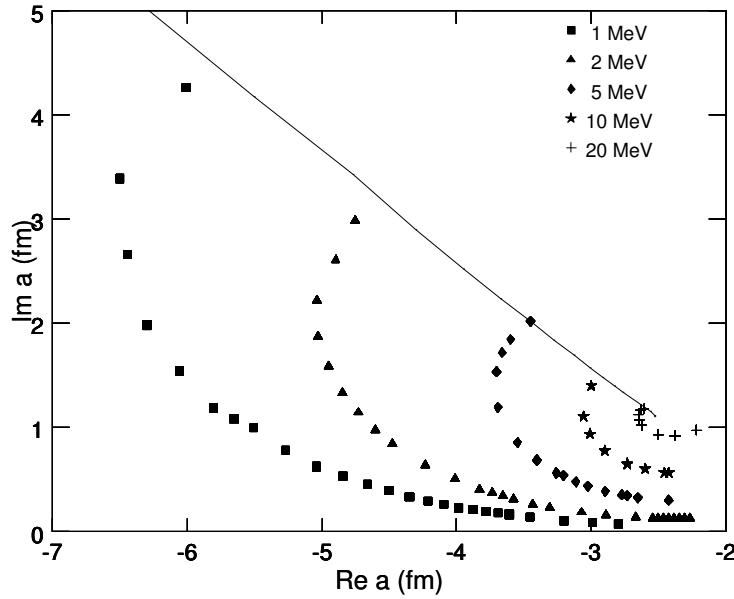


Fig. 2. The same as Fig. 1 but for the imaginary part of the binding energies E_I , *i.e.* half widths.

and E_I is anyway monotonous once the corresponding other part is kept constant. In contrast, for a given V_R a_I is *not* monotonous with respect to V_I . This behaviour results in the two branches of the E_R plots. The upper branch (starting from left for $E_R < 10$ MeV and potential real) could be considered as a "weak potential" part. A general rule of thumb is that the imaginary potential, absorption, behaves like repulsion (though the above mentioned non-monotonousness means that, in a way as inelasticity, it eats its own effect out at some stage). While E_I increases, E_R decreases, eventually to no binding. The lower "strong potential" part is particularly dictated by the repulsive effect of the imaginary part of the potential. There an even mesh of potential strengths gives an increasingly dense accumulation of points. However, this region of $|a| \approx 2-3$ fm is of most interest concerning both theoretical predictions and experiments at least in the helium case.

Therefore, Fig. 3 shows a magnified view of this region. It can be seen that with increasing binding energy the upper "weak potential" branch comes down, whereas the lower "strong potential" part slowly rises. Therefore the opening angle between the weak and strong potential branches gets smaller until at about 9 MeV binding energy they switch over. Consequently, the zero binding curve (solid) in the lower branch is not actually a limit of possible bound states (as the upper branch of the solid line is). The "weak" coupling states get below it. It is also noteworthy that the strong coupling results as curves are not very far from each other (again in contrast to the weak coupling) and consequently not far from the lower zero energy line. In fact the 1 and 2 MeV results are indistinguishable in that case. Further it may be noted that the

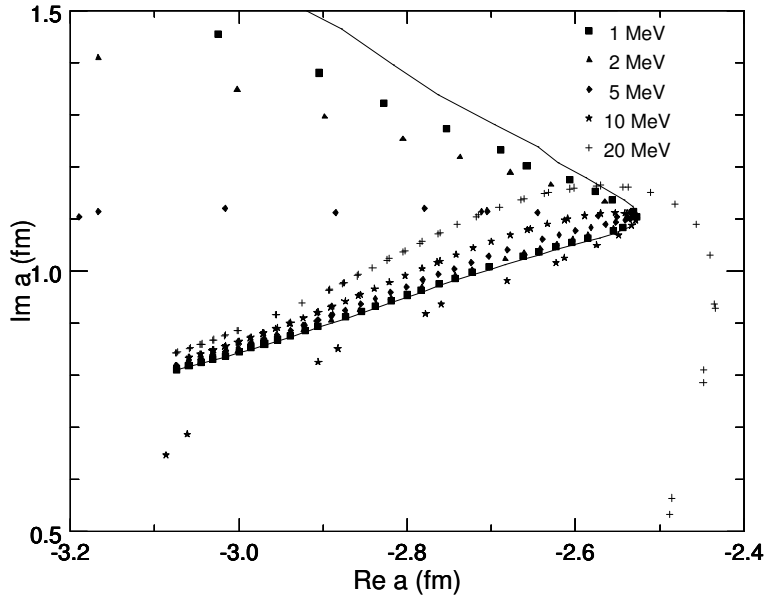


Fig. 3. Magnified detail of Fig. 1

most "eastern" point of the zero binding curve is $(-2.525 + i 1.102)$ fm.

In addition to the absolute values of binding energies and widths, of paramount experimental interest is their relative magnitude. Typically for experimental recognition of a bound state one would hope the width or half-width to be less than the binding energy for distinguishing a state from continuum. For this purpose Fig. 4 shows by points the region for which $|E_R| > |E_I|$. This belongs to the realm of "weak" coupling results. Quite clearly the real part of the scattering length in general should be larger than the imaginary part.

Carbon in most models would support binding and it is also of great interest to get predictions for lighter nuclei with possible final state interaction fits. Therefore we used the three parameter Fermi distribution [29]

$$\rho(r) = 0.06 \frac{1 + 0.517r^2/0.964^2}{1 + \exp((r - 0.964)/0.322)} \quad (6)$$

for the density profile of ^4He to get similar estimates in a much lighter and more controversial case. ^3He was studied in Ref. [28] (and the question raised again in Ref. [33]). Considering that even the nuclei are rather different, as seen in Fig. 5 the results are surprisingly similar to those of carbon. In practice only the dense accumulation point for strong potential has changed from about $(-2.5 + i 1.1)$ fm to $(-1.5 + i 0.9)$ fm. For a real potential the scattering length corresponding to 1 MeV binding changes from -7.9 fm to -7.4 fm. These, of course, reflect primarily the difference in effective ranges, which for carbon varies roughly between 2.5 fm ("weak" potential) and 1.5 fm ("strong") and

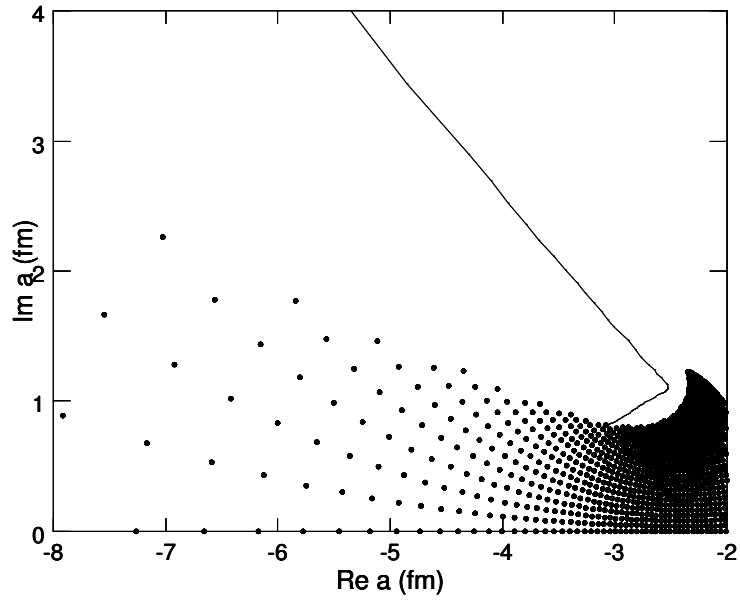


Fig. 4. The region where $|E_R| > |E_I|$.

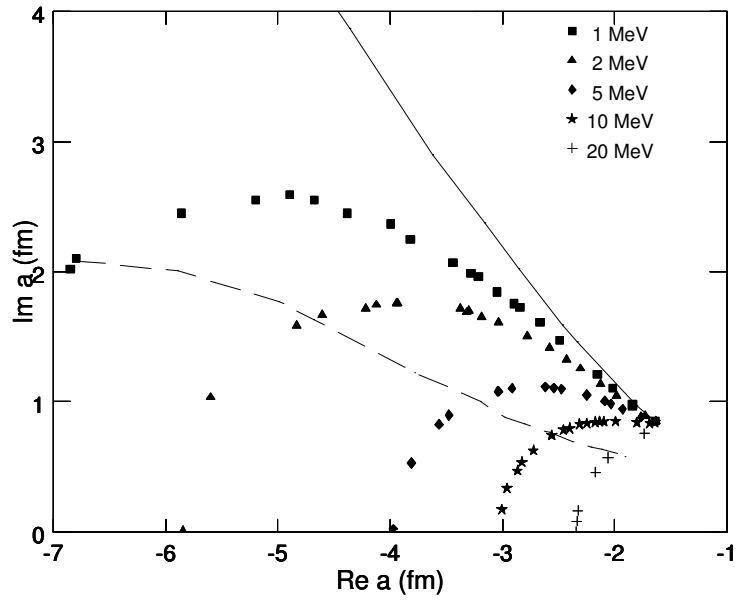


Fig. 5. The same as Fig. 1 but for the nucleus ${}^4\text{He}$. Below the dashed line $|E_R| > |E_I|$

for helium 1.7 fm and 1 fm, respectively, and is, of course, complex.

4 Conclusion

In this work a phenomenological connection between the low energy scattering length and the complex binding energy of possible eta-nuclear bound states is given in a simple but probably realistic model. The purpose of the work is that the results may be of use in searches of these bound states, if more easily accessible final state data are available to make predictions where to look for the states.

The calculations suggest that for even relatively moderate values of the imaginary potential and of the imaginary parts of the scattering lengths, the states can be wide especially compared with the real depths of the states. In view of also many other theoretical results starting from the elementary ηN scattering and predicting negative real parts for the scattering length but with rather large imaginary parts the observation of such bound states might be difficult or even impossible. However, in the minireview [12] of the situation a reanalysis of the existing data on $\eta^3\text{He}$ final states makes very small values of the imaginary plausible, so that also the possible bound states may not necessarily be as wide as most theoretical works would indicate.

In our work for a_I less than 2 fm with a_R larger than, say, 5 fm a bound state should be recognizable. In the case of more likely smaller scattering lengths $a_I < 1$ fm would be necessary. In this respect the result $|a_R| = 6.2 \pm 1.9$ fm and $a_I = 0.001 \pm 6.5$ fm of Ref. [25] is quite interesting and suggestive. The relation between a and E (as evidenced by Figs. 1 and 5) is very robust against potential differences even between different nuclides. Therefore, due to this shape independence one may trust the results to be valid also for the experimentally interesting $A = 7$ nuclei.

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